AA214A MIDTERM November 9, 1998

Instructor: Thomas H. Pulliam

In CLASS, OPEN BOOK, 25 Points + 3 Possible Bonus Points

Instructions: Show all your work. Just writing the answer will not get you credit. If you get bogged down with some algebra, write out how you would proceed to complete the problem (it will get you some credit). The points for each problem are given along with hints and helpfull information. Make sure you read the problem completely before proceeding.

1. Given the finite difference scheme for the 3^{rd} derivative

$$\delta_{xxx} u_j = \frac{-u_{j-2} + b u_{j-1} + c u_j + d u_{j+1} + u_{j+2}}{a \Delta x^3}$$

   (a) Find a, b, c, d to produce a 2^{nd} order accurate approximation **(Points:3)**

   (b) Given a, b, c, d from 1a, find (er_t) **(Points:2)**

   (c) Find the modified wave number $(k^*)^3$ (in terms of sin and/or cos). *Note*: $\partial_{xxx} e^{ikx} = -ik^3 e^{ikx}$, so for the 3^{rd} derivative define $\delta_{xxx} u_j = -i(k^*)^3 u_j$ and in this problem find $(k^*)^3$, i.e. don't try to take the cubed root **(Points:3)**

   (d) Use the result in 1c and series expansions of sin and/or cos to find and identify the order of accuracy. **(Points:2)**

2. Consider the OΔE method (*Hint: Only consider* $u' = \lambda u$)

$$u_{n+1} = u_{n-1} + h\left(2\beta_1 (u')_n + \beta_0 (u')_{n-1}\right)$$

   (a) What is the $\sigma - \lambda$ relation, in terms of β_0, β_1? **(Points:2)**

   (b) Identify the principal and any spurious roots. **(Points:2)**

   (c) Checking the principal root, what are the conditions on β_1 and β_0 that make the method:

      i. 1^{st} Order Accurate? **(Points:2)**

      ii. 2^{nd} Order Accurate? **(Points:2)**

Help: $\sqrt{1+\epsilon} = 1 + \frac{1}{2}\epsilon - \frac{1}{8}\epsilon^2 + \cdots$

3. Consider the OΔE method applied to the representative equation $u' = \lambda u + a e^{\mu t}$

$$\begin{aligned} \overline{u}_{n+1} &= u_n + h(u')_n \\ u_{n+1} &= u_n + \tfrac{1}{2} h\left(3(\overline{u}')_{n+1} - (u')_n\right) \end{aligned}$$

   (a) Identify the characteristic matrix and particular vector, $[P(E)]$ and $\overline{Q}(E)$ **(Points:3)**

   (b) Find the $\sigma - \lambda$ relation for this method. **(Points:2)**

   (c) Identify the order of accuracy from er_λ. **(Points:2)**

LOOK ON BACK!!!!!!

4. **EXTRA CREDIT PROBLEMS** True-False Questions: (List the queston number and answer T or F along with a **BRIEF ONE SENTENCE** explaination).

   A system of PDE's produces a $\lambda = \alpha + i\beta$

   (a) The resulting ODE is stable for $\alpha > 0$ **(Points:1)**

   (b) An OΔE with $\sigma = 1 + \lambda h$ and $\alpha = 0, \beta \neq 0$ is unconditionally unstable. **(Points:1)**

   Independent of λ

   (c) The Leapfrog Scheme produces the two roots:

$$\begin{aligned} \sigma_1 &= \lambda h + \sqrt{1 + (\lambda h)^2} \\ \sigma_2 &= \lambda h - \sqrt{1 + (\lambda h)^2} \end{aligned}$$

   where σ_2 is the principal root. **(Points:1)**